Holographic chiral currents in a magnetic field

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In the presence of a quark chemical potential, a magnetic field induces an axial current in the direction of the magnetic field. We compute this current in the Sakai-Sugimoto model, a holographic model which, in a certain limit, is dual to large-N_c QCD. We also compute the analogous vector current, for which an axial chemical potential is formally introduced. This vector current can potentially be observed via charge separation in heavy-ion collisions. After implementing the correct axial anomaly in the Sakai-Sugimoto model we find an axial current in accordance with previous studies and a vanishing vector current, in apparent contrast to previous weak-coupling calculations.

§1. Introduction

The Sakai-Sugimoto model[1),2)] is a realization of the gauge/gravity duality[3)] which provides a top-down approach to a holographic model of QCD. While at present no gravity dual to QCD is known, the Sakai-Sugimoto model exhibits some of the most important properties of QCD, most notably confinement and chiral symmetry breaking. Confinement is realized within a background geometry of N_c D4-branes, extending in the 3+1 dimensions of the field theory and an extra dimension x^4.[4)] This extra dimension is compactified on a circle with radius $M_{\rm KK}^{-1}$, where the Kaluza-Klein mass $M_{\rm KK}$ is a parameter of the model and sets the scale for the mass of unwanted adjoint scalars and fermions. Two solutions for the background geometry then account for confined and deconfined phases with a first-order phase transition between them at a critical temperature $T_c = M_{\rm KK}/(2\pi)$.

Fundamental chiral (and massless) fermions are described by introducing N_f D8- and $\overline{\rm D8}$-branes (here and in most applications treated as probe branes), separated in the extra dimension.[1)] The gauge symmetries on these branes account for the global chiral symmetry $SU(N_f)_L \times SU(N_f)_R$ in the associated field theory. Two qualitatively different embeddings of the flavor branes are interpreted as chirally symmetric (D8- and $\overline{\rm D8}$-branes separated) and chirally broken (D8- and $\overline{\rm D8}$-branes connected) phases. In the version of the model used here, chiral symmetry breaking and confinement are equivalent, i.e., the ground state is chirally broken if and only if it is confined (by choosing a sufficiently small distance between D8- and $\overline{\rm D8}$-branes in the extra dimension one finds a chirally broken, deconfined phase; however, from the geometry of the model it is obvious that there cannot be a chirally restored, confined phase).

Here we are interested in the evaluation of the model in the presence of a quark chemical potential and a magnetic field. It has been shown that a magnetic field gives rise to baryon number even in the chirally broken phase[5)–7)] (the resulting baryon density being homogeneous, in contrast to instanton-like baryons in the Sakai-

Sugimoto model[8]). In our context, two physical situations are of special interest. First, large magnetic fields can be found in the interior of compact stars (surface magnetic fields of magnetars being of the order of $B \lesssim 10^{15}$ G), where also the quark chemical potential is large, $\mu \sim (400 - 500)$ MeV. In this case, we may expect an axial current in the direction of the magnetic field.[9),10)] Second, large magnetic fields can be temporarily created in noncentral heavy-ion collisions, reaching values of the order of $B \sim 10^{17}$ G. In the latter case, the interplay of the magnetic field with a nonvanishing chirality N_5, i.e., a nonzero difference between the number of left- and right-handed fermions, may lead to the so-called chiral magnetic effect.[11),12)] In the chiral magnetic effect, an electric current is generated in the direction of the magnetic field, which possibly is responsible for the charge separation that has been observed at the Relativistic Heavy-Ion Collider (RHIC).[13)] The nonzero N_5 is provided by the QCD axial anomaly which relates certain gluon configurations to a change in N_5, and thus to a nonzero current on an event-by-event basis. Here and in the original weak-coupling calculations, the gluon field fluctuations are not taken into account explicitly, a nonzero N_5 is rather described by introducing an axial chemical potential μ_5. This description has to be taken with some care since N_5 is not a conserved quantity and thus strictly speaking there is no such thing as an axial chemical potential in the thermodynamic sense.

In summary, we shall be interested in the axial current in the presence of a vector chemical potential and the vector current in the presence of an axial chemical potential. More details about the following calculations can be found in Ref.[14)]

§2. Currents and anomalies

We start from the (Euclidean) action of the Sakai-Sugimoto model for one quark flavor,[2)]

$$S = S_{\rm YM} + S_{\rm CS} \,, \tag{2·1}$$

with Yang-Mills (YM) and Chern-Simons (CS) contributions

$$S_{\rm YM} = \kappa M_{\rm KK}^2 \int d^4x \int_{-\infty}^{\infty} dz \left[k(z) F_{z\mu} F^{z\mu} + \frac{h(z)}{2M_{\rm KK}^2} F_{\mu\nu} F^{\mu\nu} \right] , \tag{2·2a}$$

$$S_{\rm CS} = \frac{N_c}{24\pi^2} \int d^4x \int_{-\infty}^{\infty} dz \, A_\mu F_{z\nu} F_{\rho\sigma} \epsilon^{\mu\nu\rho\sigma} \,, \tag{2·2b}$$

with Greek indices running over $\mu, \nu, \ldots = 0,1,2,3$. Our convention for the epsilon tensor is $\epsilon_{0123} = +1$. We have used the metric functions

$$k(z) \equiv 1 + z^2 \,, \qquad h(z) \equiv (1+z^2)^{-1/3} \,, \tag{2·3}$$

and the dimensionless constant

$$\kappa \equiv \frac{\lambda N_c}{216\pi^3} \,, \tag{2·4}$$

where λ is the 't Hooft coupling. The integration is over space-time $(\tau, \mathbf{x})$ and the holographic (dimensionless) coordinate z. In the confined phase, z extends from the left-handed boundary ($z = +\infty$) over the tip of the cigar-shaped (x^4, z) subspace

($z = 0$) to the right-handed boundary ($z = -\infty$). (In the deconfined phase, the integral over connected D8 and $\overline{\text{D8}}$ branes is replaced by two separate integrals over disconnected D8 and $\overline{\text{D8}}$ branes; in this section we restrict ourselves to the confined phase, all arguments are analogous in the deconfined phase.)

The equations of motion are

$$\kappa M_{\mathrm{KK}}^2\, \partial_z[k(z)F^{z\mu}] + \kappa h(z)\partial_\nu F^{\nu\mu} = \frac{N_c}{16\pi^2} F_{z\nu}F_{\rho\sigma}\epsilon^{\mu\nu\rho\sigma}\,, \tag{2·5a}$$

$$\kappa M_{\mathrm{KK}}^2\, \partial_\mu[k(z)F^{z\mu}] = \frac{N_c}{64\pi^2} F_{\mu\nu}F_{\rho\sigma}\epsilon^{\mu\nu\rho\sigma}\,, \tag{2·5b}$$

where the second equation is obtained from varying A_z (which, in our gauge choice, has already been set to zero in the action (2·2)).

Next we introduce the chiral currents. They are defined through the variation of the on-shell action with respect to the boundary values,

$$\mathcal{J}^\mu_{L/R} \equiv -\frac{\delta S}{\delta A_\mu(x, z = \pm\infty)} = \mp \left(2\kappa M_{\mathrm{KK}}^2 k(z) F^{z\mu} - \frac{N_c}{24\pi^2}\epsilon^{\mu\nu\rho\sigma}A_\nu F_{\rho\sigma}\right)_{z=\pm\infty}\,, \tag{2·6}$$

where the first (second) term is the YM (CS) contribution. It is only the YM part of the current, i.e., the first term in eq. (2·6), which appears in the asymptotic expansion of the gauge fields:[7] from the definition (2·6) and with $k(z) = 1 + z^2$ we find

$$A_\mu(x, z) = A_\mu(x, z = \pm\infty) \pm \frac{\mathcal{J}^{L/R}_{\mu,\mathrm{YM}}}{2\kappa M_{\mathrm{KK}}^2}\frac{1}{z} + \mathcal{O}\left(\frac{1}{z^2}\right)\,. \tag{2·7}$$

One can also confirm this relation from our explicit results below.

The divergence of the currents (2·6) can be easily computed with the help of the equation of motion for A_z (2·5b). With the left- and right-handed field strengths $F^{L/R}_{\mu\nu}(x) \equiv F_{\mu\nu}(x, z = \pm\infty)$, the corresponding dual field strength tensors $\widetilde{F}^{\mu\nu}_{L/R} = \frac{1}{2}F^{L/R}_{\rho\sigma}\epsilon^{\mu\nu\rho\sigma}$, the vector and axial currents $\mathcal{J}^\mu \equiv \mathcal{J}^\mu_R + \mathcal{J}^\mu_L$, $\mathcal{J}^\mu_5 \equiv \mathcal{J}^\mu_R - \mathcal{J}^\mu_L$, and the vector and axial field strengths introduced as $F^R_{\mu\nu} = F^V_{\mu\nu} + F^A_{\mu\nu}$, $F^L_{\mu\nu} = F^V_{\mu\nu} - F^A_{\mu\nu}$, we obtain the vector and axial anomalies

$$\partial_\mu \mathcal{J}^\mu = \frac{N_c}{12\pi^2} F^V_{\mu\nu}\widetilde{F}^{\mu\nu}_A\,, \tag{2·8a}$$

$$\partial_\mu \mathcal{J}^\mu_5 = \frac{N_c}{24\pi^2}\left(F^V_{\mu\nu}\widetilde{F}^{\mu\nu}_V + F^A_{\mu\nu}\widetilde{F}^{\mu\nu}_A\right)\,. \tag{2·8b}$$

The coefficients on the right-hand side (which receive contributions from both the YM and CS parts of the currents) are in accordance with the standard field-theoretic results for N_c chiral fermionic degrees of freedom coupled to left and right chiral gauge fields.[15] The above form of the anomaly, which is symmetric in vector and axial-vector gauge fields, is called *consistent* anomaly. If left- and right-handed Weyl spinors are treated separately, this form of the anomaly arises unambiguously. In QED, however, we must require that the vector current be strictly conserved, even

in the presence of axial field strengths. As was first discussed by Bardeen,[15] this can be achieved by the introduction of a counterterm that mixes left- and right-handed gauge fields. Having even parity, Bardeen's counterterm is uniquely given by[16]

$$\Delta S = c\int d^4x(A^L_\mu A^R_\nu F^L_{\rho\sigma} + A^L_\mu A^R_\nu F^R_{\rho\sigma})\epsilon^{\mu\nu\rho\sigma}\,, \tag{2·9}$$

where c is a constant determined by requiring a strictly conserved vector current. Because this expression can be naturally written as a (metric-independent) integral over a hypersurface at $|z| = \Lambda \to \infty$ with left- and right-handed fields concentrated at the respective brane locations, ΔS can actually be interpreted as a (finite) counterterm in holographic renormalization. In particular, it does not change the equations of motion.

The contribution of Bardeen's counterterm to the chiral currents is

$$\Delta\mathcal{J}^\mu_{L/R} = \mp c\left(A^{R/L}_\nu F^{R/L}_{\rho\sigma} - A^{L/R}_\nu F^{R/L}_{\rho\sigma} + 2A^{R/L}_\nu F^{L/R}_{\rho\sigma}\right)\epsilon^{\mu\nu\rho\sigma}\,. \tag{2·10}$$

Denoting the renormalized left- and right-handed currents by $\bar{\mathcal{J}}^\mu_{L/R} \equiv \mathcal{J}^\mu_{L/R} + \Delta\mathcal{J}^\mu_{L/R}$, and $\bar{\mathcal{J}}^\mu \equiv \bar{\mathcal{J}}^\mu_R + \bar{\mathcal{J}}^\mu_L$, $\bar{\mathcal{J}}^\mu_5 \equiv \bar{\mathcal{J}}^\mu_R - \bar{\mathcal{J}}^\mu_L$, we find that $c = N_c/(48\pi^2)$ is required to obtain the *covariant* anomaly

$$\partial_\mu\bar{\mathcal{J}}^\mu = 0\,, \tag{2·11a}$$

$$\partial_\mu\bar{\mathcal{J}}^\mu_5 = \frac{N_c}{8\pi^2}F^V_{\mu\nu}\widetilde{F}^{\mu\nu}_V + \frac{N_c}{24\pi^2}F^A_{\mu\nu}\widetilde{F}^{\mu\nu}_A\,. \tag{2·11b}$$

The prefactor in front of the first term in the axial anomaly now has changed from $N_c/(24\pi^2)$ in eq. (2·8b) to $N_c/(8\pi^2)$, which is the well-known result for the Adler-Bell-Jackiw anomaly for QED[17),18)] and which is essential for getting the correct pion decay rate $\pi^0 \to 2\gamma$. The necessity of adding the counterterm (2·9) to the Sakai-Sugimoto model is in fact completely analogous to the well-known procedure in chiral models where a Wess-Zumino-Witten term accounts for the anomaly.[19)]

In the literature sometimes the coefficient of the subleading term in the asymptotic behavior of $A_\mu(z)$ and thus the YM part of the current (see eq. (2·7)) is identified with the full current.[20),21)] Using this identification, it has also been assumed that the equation of motion for A_z (2·5b) represents the anomaly equation.[22)] Indeed, from eq. (2·5b) one obtains

$$\partial_\mu\mathcal{J}^\mu_{\rm YM} = \frac{N_c}{4\pi^2}F^V_{\mu\nu}\widetilde{F}^{\mu\nu}_A\,, \tag{2·12a}$$

$$\partial_\mu\mathcal{J}^\mu_{\rm YM,5} = \frac{N_c}{8\pi^2}\left(F^V_{\mu\nu}\widetilde{F}^{\mu\nu}_V + F^A_{\mu\nu}\widetilde{F}^{\mu\nu}_A\right)\,, \tag{2·12b}$$

and this does contain the same coefficient in front of $F^V_{\mu\nu}\widetilde{F}^{\mu\nu}_V$ as the full covariant anomaly (2·11). However, it differs from the latter in the presence of axial gauge fields. In particular, the vector current is then not strictly conserved. Even when this issue may be ignored, because all axial vector field strengths are set to zero, it appears to be questionable to keep only part of the full current (2·6).

§3. Solution to the equations of motion

We introduce a homogeneous magnetic field B in the spatial 3-direction via the gauge field component $A_1(x_2) = -x_2 B$, i.e., A_1 is a constant with respect to the holographic coordinate z, which is consistent with the equations of motion. Then, the only nontrivial gauge field components we need are $A_0(z)$ and $A_3(z)$, where A_0 is necessary to account for finite chemical potentials, and A_3 becomes nonzero through the equations of motion. (With only a little more effort, the equations of motion can also be solved in the presence of an additional electric field.[14] This is for instance useful to check the axial anomaly explicitly, but for simplicity we restrict ourselves to a magnetic field which is the physically relevant field in our context.)

3.1. *Chirally broken phase*

Within this ansatz, the equations of motion read

$$\partial_z(k\partial_z A_0) = 2\beta\partial_z A_3\,, \qquad \partial_z(k\partial_z A_3) = 2\beta\partial_z A_0\,, \tag{3·1}$$

with the dimensionless magnetic field $\beta \equiv \alpha B/M_{\rm KK}^2$ where $\alpha \equiv 27\pi/(2\lambda)$. The general solutions to Eqs. (3·1) are

$$A_0(z) = \mu - \mu_5\frac{\sinh(2\beta\arctan z)}{\sinh\beta\pi} - \jmath\left[\frac{\cosh(2\beta\arctan z)}{\sinh\beta\pi} - \coth\beta\pi\right]\,, \tag{3·2a}$$

$$A_3(z) = -\mu_5\left[\frac{\cosh(2\beta\arctan z)}{\sinh\beta\pi} - \coth\beta\pi\right] - \jmath\,\frac{\sinh(2\beta\arctan z)}{\sinh\beta\pi}\,. \tag{3·2b}$$

Here we have identified the boundary values of the temporal components of the gauge fields with the chemical potentials, $A_0(z=\pm\infty) = \mu_{L,R}$, and the vector and axial chemical potentials are $\mu \equiv (\mu_R + \mu_L)/2$ and $\mu_5 \equiv (\mu_R - \mu_L)/2$. The boundary values of the spatial components of the gauge fields are $A_3(z=\pm\infty) = \mp\jmath$. The axial supercurrent $\jmath$, which has to be determined from minimizing the free energy, describes a rotation of the chiral condensate in the space spanned by the scalar and pseudoscalar mesons[7] and thus may be related to the recently discussed quarkyonic chiral spirals.[23]

3.2. *Chirally restored phase*

In the case of disconnected D8 and $\overline{\rm D8}$ branes, i.e., in the deconfined, chirally restored phase, we have two sets of equations of motion, one for the left-handed fields $A_\mu^L(z)$ and one for the right-handed fields $A_\mu^R(z)$ with $z \in [0,\infty]$,

$$\partial_z(k_0\partial_z A_0^{L/R}) = \pm\frac{2\beta}{\theta^3}\partial_z A_3^{L/R}\,, \qquad \partial_z(k_3\partial_z A_3^{L/R}) = \pm\frac{2\beta}{\theta^3}\partial_z A_0^{L/R}\,. \tag{3·3}$$

Here $\theta \equiv 2\pi T/M_{\rm KK}$ is the dimensionless temperature, and $k_0(z) \equiv \frac{(1+z^2)^{3/2}}{z}$, $k_3(z) \equiv z(1+z^2)^{1/2}$ are different metric functions for temporal and spatial components of the gauge fields. The general solution to Eqs. (3·3) is

$$A_0^{L/R}(z) = (\mu \mp \mu_5)\left[p(z) - \frac{p_0}{q_0}\,q(z)\right]\,, \tag{3·4a}$$

$$A_3^{L/R}(z) = \pm\frac{\mu\mp\mu_5}{2\beta/\theta^3}\left[k_0\partial_z p - \frac{p_0}{q_0}(1+k_0\partial_z q)\right] \, , \qquad (3\cdot4b)$$

where $p_0 \equiv p(0)$, $q_0 \equiv q(0)$, and

$$p(z) = {}_2F_1\left[-\frac{\sqrt{1-16\beta'^2}+1}{4}, \frac{\sqrt{1-16\beta'^2}-1}{4}, \frac{1}{2}, \frac{1}{1+z^2}\right] \, , \qquad (3\cdot5a)$$

$$q(z) = \frac{1}{\sqrt{1+z^2}}\,{}_2F_1\left[-\frac{\sqrt{1-16\beta'^2}-1}{4}, \frac{\sqrt{1-16\beta'^2}+1}{4}, \frac{3}{2}, \frac{1}{1+z^2}\right] \, , \quad (3\cdot5b)$$

with the abbreviation $\beta' \equiv \beta/\theta^3$. Again, the boundary values of the temporal components of the gauge fields are identified with the chemical potentials, $A_0^{L/R}(\infty) = \mu_{L/R}$. Following Ref.,[24] these components vanish at the horizon, $A_0^{L/R}(0) = 0$ (see however Refs.[25),26)]). The spatial components vanish at the holographic boundary, but acquire a finite value at the horizon, $A_3^{L/R}(0) = \mp\frac{\mu_{L/R}}{2\beta/\theta^3}\frac{p_0}{q_0}$.

§4. Results and discussion

To compute the axial and vector currents in the direction of the magnetic field we use the solutions to the equations of motion, Eqs. (3·2) and (3·4), and insert them into the chiral currents, which are defined through Eq. (2·6) and the contribution from Bardeen's counterterm (2·10). The results are summarized in Table I.

Let us first discuss the full renormalized currents $\bar{\mathcal{J}} = \mathcal{J}_{\rm YM} + \mathcal{J}_{\rm CS} + \Delta\mathcal{J}$ which have been obtained under the constraint of the correct (covariant) QED anomaly. For the axial current we find that the counterterm $\Delta\mathcal{J}$ exactly cancels the CS part. In the chirally symmetric phase, this yields exactly the expected topological result $\mathcal{J}_5 = \frac{\mu B N_c}{2\pi^2}$.[9),10)] In particular, the dimensionful parameter of the model, $M_{\rm KK}$, drops out of this universal result. In the chirally broken phase, the axial current is smaller and given by a complicated function of B.

The most striking of our results is that for both phases the renormalized vector current is zero for all magnetic fields. One might have expected that, in the deconfined phase, this current should reproduce the known result for the chiral magnetic effect,[12)] $\mathcal{J} = \frac{\mu_5 B N_c}{2\pi^2}$, because this result can be derived from the anomaly only[27)] and we have explicitly made sure that our model reproduces the correct anomaly. The question is thus whether our result can indeed be interpreted as the current responsible for the chiral magnetic effect. We shall comment on possible issues related to this question below. Our result of a vanishing current in the confined phase, however, seems less puzzling. The usual explanation of the chiral magnetic effect, using a quasiparticle picture (which is not guaranteed to hold in our strong-coupling approach), relies on individual, electrically charged, massless quarks which move in different directions according to their chirality. A suppression of the effect may thus indeed be expected in the confined, chirally broken phase.[12),28)]

In Table I we also show the results which are obtained from the YM part only. In the case of the axial currents, there is no difference to the renormalized currents. For

		$\mathcal{J}_{\rm YM}$	$\mathcal{J}_{\rm YM}+\mathcal{J}_{\rm CS}$	$\mathcal{J}_{\rm YM}+\mathcal{J}_{\rm CS}+\Delta\mathcal{J}$
	anomaly	"semi-covariant":	consistent:	<u>covariant:</u>
	$\partial_\mu\mathcal{J}_5^\mu/\frac{N_c}{24\pi^2}$	$\underline{3F_V\widetilde{F}_V}+3F_A\widetilde{F}_A$	$F_V\widetilde{F}_V+F_A\widetilde{F}_A$	$\underline{3F_V\widetilde{F}_V+F_A\widetilde{F}_A}$
	$\partial_\mu\mathcal{J}^\mu/\frac{N_c}{24\pi^2}$	$6F_V\widetilde{F}_A$	$2F_V\widetilde{F}_A$	$\underline{0}$
$T>T_c$	$\mathcal{J}_5/\frac{\mu B N_c}{2\pi^2}$	1	$\frac{2}{3}$	1
	$\mathcal{J}/\frac{\mu_5 B N_c}{2\pi^2}$	1	$\frac{2}{3}$	0
$T<T_c$	$\mathcal{J}_5/\frac{\mu B N_c}{2\pi^2}$	$\frac{\beta\coth\beta\pi}{2\rho(\beta)}$	$\frac{\beta\coth\beta\pi}{2\rho(\beta)}-\frac{1}{3}$	$\frac{\beta\coth\beta\pi}{2\rho(\beta)}$
	$\mathcal{J}/\frac{\mu_5 B N_c}{2\pi^2}$	1	$\frac{2}{3}$	0

Table I. Different (parts of the) axial and vector currents $\mathcal{J}_5$ and $\mathcal{J}$ – normalized to the weak-coupling results[9),12)] – in the direction of the magnetic field B in the deconfined, chirally symmetric phase ($T>T_c$) and in the confined, chirally broken phase ($T<T_c$). We have abbreviated $\rho(\beta)\equiv\beta\coth\beta\pi+\frac{\pi\beta^2}{2\sinh^2\beta\pi}$ with the dimensionless magnetic field $\beta=\alpha B/M_{\rm KK}$.

the vector currents, however, we now recover the weak-coupling (deconfined) result for both confined and deconfined phases. Therefore, now the deconfined current is as expected while the lack of suppression in the confined phase comes as a surprise.

There are several problems in the current approach. First, upon computing the free energy explicitly and then taking the derivative with respect to the appropriate source, the currents turn out to be different from the straightforward definition via the gauge/gravity correspondence (used for the results in Table I). This disturbing discrepancy arises for nonvanishing background magnetic fields and can be attributed to boundary terms at spatial infinity. A previously suggested fix of this problem by modifying the action[5)] seems to be not acceptable for our purpose because it entirely eliminates the axial anomaly from the correspondingly modified currents.[14)] Second, the introduction of a μ_5 may be problematic. Since a chemical potential must be associated to a conserved charge, it has been argued that a physically meaningful vector current can be obtained only after introducing a conserved (not anomalous) charge N_5.[29),30)] This, on the other hand, corresponds to a gauge *variant* axial charge density, which precludes a generalization to inhomogeneous situations.[26)]

In a simple holographic model not unlike the Sakai-Sugimoto model, and in a setup that corresponds to our deconfined phase, the vector current has recently been found to coincide with the weak-coupling result.[26)] In this study, which uses linear response theory (i.e., only deals with an infinitesimally small magnetic field), it has been pointed out that it is important to distinguish the source for the quark density (= boundary value of the gauge field) from the chemical potential (= potential difference between boundary and horizon). With this distinction the sources can be set to zero, and consequently the CS contributions to the currents vanish trivially. It remains to be seen whether this concept can be applied to the present model, in particular to the case of a finite background magnetic field.[31)] Moreover, the situation in the confined phase seems profoundly different, since in the case of connected branes it is unclear how to define the chemical potential other than through the

boundary value. Maybe this difference is not surprising because it is the deconfined phase in which the results of the renormalized currents seem puzzling and the YM currents seem to agree with physical expectations. In the confined phase it is the other way around.

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